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Markowitz's Portfolio Selection Theory in Upstream Oil and Gas Investment Decision: A Mean-Variance Optimization Approach

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Abstract

Background: The upstream oil and gas industry is characterized by high capital intensity, oil price volatility, and geological uncertainty, making investment decisions challenging. Despite the growing adoption of Markowitz's Modern Portfolio Theory (MPT) in financial markets, its empirical application to upstream oil and gas investment remains limited.
Objectives: This study assesses the utility of Markowitz's mean-variance portfolio selection theory for upstream oil and gas investment decision-making using historical monthly returns from five major energy companies.
Methods: A descriptive quantitative approach was employed using secondary data from Yahoo Finance, adjusted monthly closing prices from January 2018 to January 2024. Risk optimization used variance-covariance matrices, efficient frontier construction, and Monte Carlo simulation to address non-symmetrical return distributions.
Results: Portfolio analysis revealed a high correlation between BP and Shell, approximately 0.92. Key indicators showed cyclicality, with downturns in 2015, 2016, and 2020, followed by rebounds in 2021 and 2022. Persistent upstream capital expenditure was observed for HESS and EQNR, while BP showed volatile profitability in 2022. CVX and Shell showed moderate returns on investment and assets. The efficient frontier and Sharpe ratio indicated excess returns of approximately 3.7–3.8% per unit of volatility above the risk-free rate, affecting combinations. These portfolios were compared with an equal-weight benchmark, which may not represent an efficient allocation.
Conclusion: Markowitz's portfolio framework provides a rigorous and practical tool for upstream oil and gas investment decisions. However, optimal portfolio composition depends on risk definition and efficient frontier characteristics, offering actionable implications for energy portfolio managers balancing risk and return.

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INTRODUCTION

The upstream oil and gas industry represents one of the most capital-intensive sectors in the global economy (Abdelhamid et al., 2021; Abor et al., 2023; Osei et al., 2023). In 2023, global upstream oil and gas investment exceeded USD 530 billion, with continued exposure to oil price volatility, exploration risk, and energy transition pressures. Upstream projects are characterized by large, irreversible capital outlays, long payback periods, and highly uncertain cash flows driven by reserve uncertainty, production variability, and commodity price fluctuations.

These features demand rigorous portfolio management tools that go beyond single-project net present value (NPV) analysis. The global financial community widely recognizes Markowitz as the father of Modern Portfolio Theory (MPT). Markowitz (2008) proposed the mean-variance framework, which revolutionized investment analysis by providing a quantitative basis for constructing portfolios that maximize returns for a given level of risk. His work emphasizes the covariance structure of asset returns and demonstrates that diversification can reduce portfolio risk without sacrificing expected return. Despite subsequent theoretical advances, Markowitz's principles remain foundational to how portfolio managers make decisions today.

Markowitz introduced the portfolio mean-variance model in 1952, marking a significant step in quantitative finance by helping investors balance the potential returns of their investments against their risks. In 1964, Sharpe (1) further developed this framework by introducing the Capital Asset Pricing Model (CAPM), which provides a clearer explanation of how asset returns relate to risk. Subsequently, Ross introduced Arbitrage Pricing Theory (APT) in 1976, offering an alternative perspective on risk and return without some of the strict assumptions of CAPM. This development led to further advancements in financial theory, including models that account for various factors influencing investment performance (Al-Harthy, 2007). In recent decades, financial theory has evolved significantly since the introduction of the Markowitz model, which focuses on building investment portfolios that balance risk and return. Notable advancements include the Intertemporal Capital Asset Pricing Model, proposed by Robert Merton, which considers how investors make decisions across multiple time periods Brashear (2000), and the Consumption Capital Asset Pricing Model, developed by Martin Breeden, which links asset prices to consumer behavior and consumption over time. These developments contribute to a more comprehensive understanding of capital markets and asset valuation (Bodie et al., 2013).

Empirical applications of MPT in upstream oil and gas have grown since the early 1990s. Hightower (1991) first introduced portfolio modeling to the sector, while Xue (2014) and Rodriguez (2012) demonstrated improved capital allocation using mean-variance optimization for overseas upstream assets. Despite these contributions, three important gaps remain unaddressed: (1) the inconsistency between financial risk definitions, namely the variance of returns, and oilfield risk terminology, namely the probability of loss; (2) insufficient empirical validation using recent energy company data; and (3) limited guidance on how different risk definitions affect optimal portfolio selection. The present study addresses these gaps by applying MPT to a portfolio of major listed energy companies using historical data.

Conducted studies focusing on investment risk analysis in the oil industry, particularly in upstream oil and gas projects (Wang & Wang, 2008). These projects are characterized by large capital requirements, long investment horizons, and significant uncertainty, making risk assessment a critical component of investment decision-making. Appropriate investment decisions can generate substantial returns, whereas poor decisions may lead to considerable financial losses. Consequently, identifying, evaluating, and managing risks are essential for reducing uncertainty and improving project performance (Hillson, 2024).

The novelty of the present study lies in its integration of variance-based and probability-based efficient frontier approaches with Monte Carlo simulation to evaluate investment portfolio performance in the upstream oil and gas sector. By comparing these alternative risk definitions within a unified analytical framework, the study demonstrates how different risk measures can significantly influence optimal portfolio composition and investment outcomes. Furthermore, the research aims to provide a practical, replicable, and data-driven framework that can support practitioners and decision-makers in developing more effective energy investment portfolio strategies under conditions of uncertainty.

METHOD

This study employed a quantitative approach to analyze the application of Modern Portfolio Theory in investment decision-making within the upstream oil and gas sector. This approach was selected because it enabled the objective measurement of the relationship between risk and return through mathematical models and statistical analysis. The research adopted a descriptive quantitative design with a portfolio analysis framework, focusing on the measurement of expected return, risk, including variance and standard deviation, and portfolio optimization using the efficient frontier concept (Zanjirdar, 2020).

The data used in this study were secondary data obtained from multiple sources, including historical stock return data of global oil and gas companies, relevant academic literature on portfolio theory, and financial data platforms such as Yahoo Finance. The research objects consisted of major energy companies, including Chevron, BP, Shell, Hess, and EQNR, which were selected to represent the characteristics of a high-risk and high-volatility industry.

The research instruments consisted of mathematical portfolio models, including the calculation of expected portfolio return, risk measurement using variance and covariance, and optimization models based on the efficient frontier. Data collection techniques were conducted through documentation and literature study, involving the collection of historical market data and relevant academic references.

The data analysis proceeded through five stages. First, monthly log returns were computed from adjusted closing prices for each security. Second, expected portfolio return was calculated as the weighted average of individual asset mean returns. Third, portfolio risk was measured using a full variance-covariance matrix derived from 36-month and 60-month rolling windows. Fourth, the efficient frontier was constructed by solving the classical Markowitz mean-variance optimization problem, which minimized portfolio variance subject to a target expected return constraint and portfolio weight constraints, with weights summing to 1 and short-selling permitted within the range of −100% to +200%. Fifth, Monte Carlo simulation, consisting of 10,000 portfolio weight draws from a uniform random distribution with constraint enforcement, was employed to generate a cloud of feasible portfolios and identify the efficient frontier under non-normal return distributions. All calculations were implemented in Microsoft Excel, with manual matrix algebra verification. The statistical assumptions included return stationarity within each sub-period and normally distributed returns for the variance-based analysis, although the latter assumption was relaxed in the Monte Carlo extension.

The research procedure began with the collection of historical data, followed by the calculation of statistical parameters such as mean, variance, and covariance. Subsequently, portfolio models were developed, asset combinations were analyzed, and optimal portfolios were determined. Finally, the results were interpreted to draw conclusions. This methodological framework was adapted from the analyzed study and refined to meet systematic and structured academic writing standards.

RESULTS AND DISCUSSION

Results

The Measurement Method of the Portfolio Return

This section presents the empirical results of applying Markowitz's mean-variance portfolio theory to selected oil and gas companies. The subsections cover portfolio return measurement, portfolio risk measurement, the efficient frontier model, and a case study application using data through December 2006. In the traditional Markowitz portfolio selection model, portfolio return is calculated as the weighted average of the returns of each asset included in the portfolio. This method is applicable to the oil and gas industry, but there are key differences in upstream oil and gas investments. Unlike stock investments, in which capital can be divided easily and some funds can remain unused, oil and gas investment projects involve fixed assets that must either be included in or excluded from the portfolio. This creates a binary investment choice and often results in surplus funds that are not fully invested. Therefore, the expected return of a portfolio containing N assets can be expressed as Eq. 1:

$$E(R_P) = W_0R_0 + \sum_{i=1}^{N} X_iW_i\, R_i(i = 1,2 \dots , n) \qquad (1)$$

where E(Rp) denotes the expected return of the portfolio; W_0 denotes the proportion of surplus funds in the total investment capital; R_0 denotes the bank interest rate; X_i is a binary decision variable equal to 1 if the i-th project is included in the portfolio and 0 otherwise; W_i denotes the proportion of the i-th asset in the total investment capital, with $\sum_{i=1}{}^{N} W_i = 1$ and $W_i \geq 0$; R_i denotes the expected return of the i-th asset; and N denotes the number of assets in the portfolio.

The measurement method of the Portfolio risk

In the Markowitz portfolio selection model, portfolio risk is commonly measured using variance or standard deviation, which indicates the extent to which investment returns may deviate from their expected values. Essentially, if the actual returns of an asset frequently differ substantially from the expected returns, this indicates a higher level of risk. Therefore, understanding these variations helps investors assess the risk associated with their investments. Portfolio risk can be expressed as follows in

Eq. (2):

$$V=\sigma_p^2=\sum_{i=1}^{N} W_i^2\,\sigma^2(R_i)+2\sum_{i=1}^{N}\sum_{j=1}^{N} W_iW_j \operatorname{cov}\left(R_i,R_j\right)(i\neq j) \quad (2)$$

Where $\sigma^2(R_i)$ the variance of the expected return for the i-th asset; $\operatorname{cov}(R_i, R_j)$ covariance between i-th asset and j-th asset (or correlation coefficient).

Based on the equation above, portfolio risk is influenced by both the individual risk of each asset in the portfolio and the relationship among the assets, as measured by covariance or the correlation coefficient. A diversified investment portfolio, meaning one that consists of various assets, does not necessarily increase expected returns, but it can reduce fluctuations in returns and thereby lower overall investment risk. The objective of effective portfolio management is to select assets with low correlations to minimize risk while still targeting a certain level of expected return.

In the oil and gas industry, risk measurement using variance, which generally assumes a relatively symmetric distribution of outcomes, can be misleading, particularly in relation to investment returns and asset values. Returns from upstream oil and gas exploration often exhibit skewed distributions, indicating a higher probability of extreme outcomes, either positive or negative. Therefore, this paper discusses the use of Monte Carlo simulation to evaluate alternative risk definitions and generate an efficient frontier that can support more informed investment decision-making.

Mathematical Model of Portfolio Efficient Frontier

The mathematical model of the portfolio efficient frontier represents a set of optimal investment strategies that aim to achieve the highest expected return for a given level of risk or the lowest risk for a specified level of return. It involves balancing multiple objectives and constraints, such as total investment cost and minimum production requirements, to determine the best combination of assets. By using techniques such as genetic algorithms, investors can identify portfolios that maximize returns while minimizing risk, thereby supporting more effective investment decisions.

In the context of portfolio management, the efficient frontier represents the optimal combination of investments that maximize returns while minimizing risk. The underlying assumptions state that investors prefer higher potential returns when faced with similar levels of risk and will choose lower risk when expected returns are the same. Portfolios that meet these criteria are called "efficient sets," and a mathematical model is developed to ensure that these conditions and other constraints are satisfied in determining optimal investment choices. The mathematical model in Eq. (3) is established to satisfy the following constraints:

$$\text{s.t}\quad \begin{cases} V=\sum_{i=1}^{N}\sum_{j=1}^{N} W_i\, W_j \operatorname{Cov}\left(R_i,R_j\right) \\ C=\sum_{i=1}^{n} C_i \leq C_a \\ P=\sum_{i=1}^{n} P_i \geq P_a \\ W_0+\sum_{i=1}^{N} X_i\, W_i=1,\ W_i\geq 0,\ \text{i-1,2,...N} X_i=0 \text{ or } 1 (i=1,2,\ldots N) \end{cases} \quad (3)$$

where C is the total investment capital of the optimal portfolio; Ca is the maximum investment cost constant; P is the equivalent production of the optimal portfolio; Pa is the minimum equivalent production constant; and n is the number of assets in the optimal portfolio.

In multi-objective planning for investment portfolios, a mathematical model may include various limits or constraints based on real-world conditions. These constraints may specify the maximum investment cost, minimum production requirements, or cash flow limits for individual projects. To optimize asset selection within these limits, genetic algorithms can be used to identify the best combination of investments that satisfies both financial objectives and practical restrictions. Finally, the genetic algorithm is applied to optimize the selection of stochastic assets and determine an optimal portfolio set that satisfies both the objectives and the constraints.

Example of Application

This study examines the performance of a selected group of five major oil and gas companies as a case study: Chevron (CVX), BP p.l.c. (BP), Shell, Hess, and EQNR. These companies operate globally in both offshore and onshore activities, extracting oil and gas from various locations around the world. By analyzing their stock performance, this study provides insight into how different companies may be affected by factors such as oil prices, which are important in constructing an optimal investment portfolio.

Intuitively, it can be inferred that the returns of BP and SHELL are likely to be closely linked, suggesting that their correlation coefficient would approximately 1.0. Both corporations have wholly owned subsidiaries engaged in exploration and production, as well as refining and marketing. Consequently, fluctuations in oil and gas commodity prices should have a similar impact on both companies.

When commodity prices rise, the revenues of both BP and SHELL increase; conversely, when prices decline, their revenues decrease. However, it is also reasonable to suggest that the returns of HESS and EQNR may not exhibit a high degree of correlation, leading to a correlation coefficient that significantly diverges from 1.0. Unlike HESS, rising oil and gas commodity prices may represent an increase in costs for EQNR, while for EQNR, they signify an increase in revenue. Similarly, a decrease in commodity prices may reduce BP's costs, whereas for EQNR, it may indicate a decline in revenue.

Table 1 illustrate the monthly return calculation for each of the five securities and how these returns average and vary over a five-year time horizon as measured through December 2018. Based on this analysis over the past three years, Shell has an average monthly return of 2.86% with a standard deviation of 5.76%; CVX has an average monthly return of 5.95% with a standard deviation of 34%; BP has an average monthly return of 1.85 % with a standard deviation of 35.5%.

The measured of **covariance matrix** $\left(\Sigma = \frac{X^T X}{n-1}\right)$ for the returns of 5 securities (CVX, BP, SHELL, HESS, EQNR) visualized in Table 3 where each entry indicates how the returns of a pair of securities move together; higher off-diagonal values imply stronger co-movement (risk relationship), which is crucial for calculating portfolio risk and optimizing diversification. The covariance matrix summarizes how each pair of securities' returns move together: diagonals are individual variances (risk of each stock), and off diagonals are covariances (co-movement between two stocks). $\Sigma = X^T X/(n-1)$ means you take the matrix of mean-centered return observations (X), multiply by its transpose, and scale by (n−1) to get unbiased estimates. In portfolio optimization, Σ is used to compute total portfolio variance given weights, guiding construction of the Efficient Frontier and Sharpe ratio maximization. Each diagonal entry (e.g., 0.00046 for CVX) is the variance of that security's returns, and each off-diagonal entry (e.g., 0.00038 between CVX and BP) is the covariance, indicating how two securities' returns move together.

This matrix is essential for computing portfolio variance/standard deviation and thus for plotting the efficient frontier and optimizing the risk-return tradeoff. Another situation with the analysis for example HESS Covariance number 0.00095, where numeric result from one of the portfolio calculations e.g., a daily expected return, a variance term from the covariance matrix, or a small portfolio weight. At this scale, 0.00095 as a daily return equals about 0.095% per day (roughly 24% annualized if 252 trading days), or as a variance it implies a standard deviation of sqrt (0.00095) ≈ 3.1% over the same period. Interpreting it correctly depends on the cell's context (units and whether it's a return, variance, or weight) in the Excel workflow.

However, returns, variance, and covariance are not constant over time. Over the five years ending December 2023, CVX's average monthly return is 1.46% with a standard deviation of 5.62%; CVX's average monthly return is 1.28% with a standard deviation of 5.65%; and HESS's average monthly return is 0.01% with a standard deviation of 8.00%.

This table is the correlation matrix for five stocks (CVX, BP, SHELL, HESS, EQNR). Each entry is the correlation coefficient $\rho ij = Cov(i,j)/(\sigma i\ \sigma j)$, summarized as $(\Sigma/(\sigma^T\sigma))1/2$ in matrix form: Σ is the covariance matrix, σ is the vector of standard deviations, and normalizing by σ scales covariances into correlations between −1 and 1. Values near 1 (e.g., 0.90 between CVX and HESS) mean the stocks tend to move together strongly; 1.000 on the diagonal is self-correlation. The correlation **coefficients** between the returns of five securities (CVX, BP, SHELL, HESS, EQNR), where each value measures how similarly two stocks move together, ranging from −1 (perfectly negative) to 1 (perfectly positive), with the diagonal always being 1 (since a stock is perfectly correlated with itself). These correlations are key in **portfolio optimization** because lower correlations among assets can reduce portfolio risk through diversification. E.g., 0.8340 between CVX and BP shows how strongly two assets move together (1 = perfectly together, 0 = no linear relation), and the diagonal is 1.0000 because each asset is perfectly correlated with itself. In portfolio optimization, this matrix is used to build the covariance matrix (by multiplying correlations by individual standard deviations), which then determines portfolio risk and the Efficient Frontier. Another situation the highest correlation number for example 0.9175 (BP and Shell) is likely a computed metric from the portfolio analysis in Excel most plausibly a Sharpe Ratio, correlation, or a portfolio weight/expected return figure. If it's a Sharpe Ratio, 0.9175 means the portfolio delivers about 0.92 units of excess return per unit of risk, which is moderate efficiency. If it's a correlation or covariance-derived value, 0.9175 indicates a very strong positive relationship between two securities, affecting diversification and the Efficient Frontier shape.

Figure 1 supports the previous expectations regarding these five portfolios, indicating that firms within the same industry tend to exhibit higher correlations in security performance, particularly when the revenue drivers of one industry serve as the cost drivers of another.

By combining two investments with correlation coefficients below 1.0, it is possible to create a portfolio with reduced variability for a specified level of return. Table 2, along with Figures 1 and 2, illustrates the decrease in variability for asset combinations of HESS and CVX, as well as HESS and SHELL. Furthermore, Table 2 uses the returns and variances calculated over the most recent three years for the three securities.

Different weighting factors are applied to HESS in relation to CVX and to HESS in relation to BP. These weighting factors range from −100% to 200%, ensuring that the sum of the two component weights equals 100%. This approach facilitates the assessment of a short position in one security relative to the other during the evaluated time frame.

The portfolio's standard deviation is denoted as St. Dev. (Pf), while the expected return of the portfolio is represented as Expected Return (Pf). To illustrate, when the portfolio consists of 50% HESS and 50% CVX, the expected return is 1.99%, with a standard deviation of 5.22%. In contrast, for a portfolio comprising 50% HESS and 50% EQNR, the expected return is 1.09%, while the standard deviation is 4.17%.

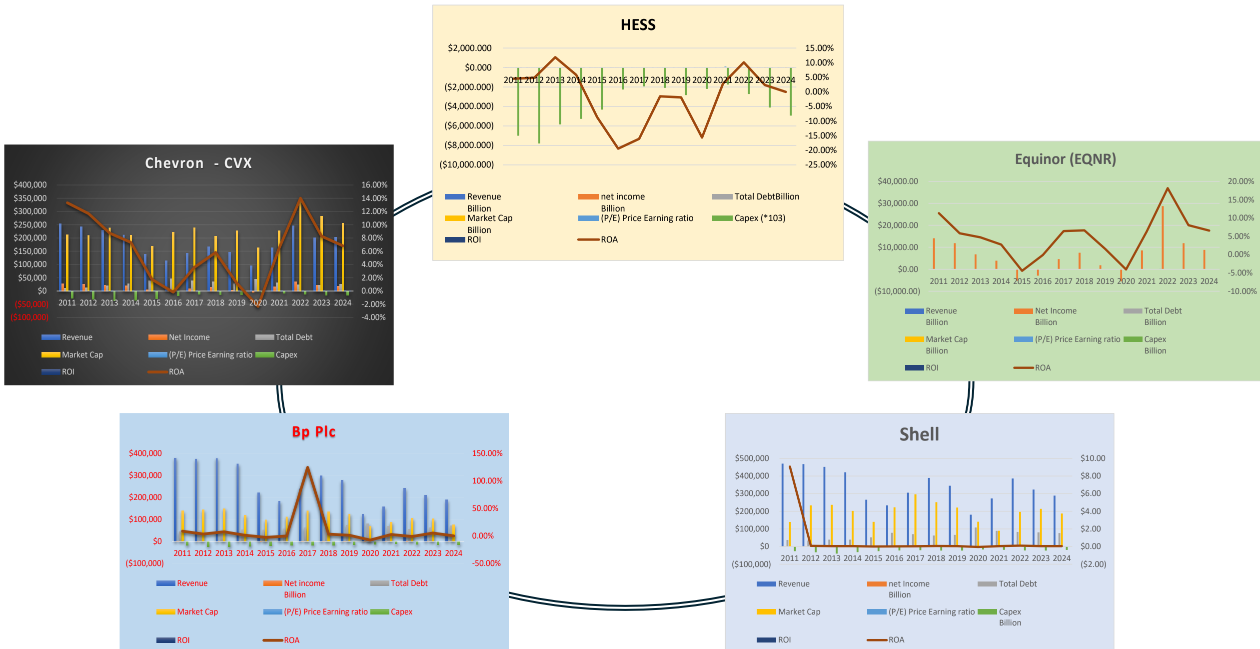

Figure 1. Global portfolio market competition in the upstream oil and gas industries

These series of graphical financial analysis showing a multi-metric, year-by-year view of major oil and gas firms Chevron, BP, Shell, Hess, and EQNR using standard finance measures to assess scale (Revenue, Market Cap), profitability (Net Income, ROI, ROA), leverage (Total Debt), valuation (P/E), and reinvestment (Capex). For Hess (2011–2024), the chart's axes and legend indicate how these series move over time, letting you see cycles tied to oil prices (e.g., revenues/profits rising and falling), balance-sheet changes (debt), and how markets value the firm (P/E) relative to its earnings. Interpreting them together reveals whether growth is being funded with debt or cash flows, how efficiently assets generate returns, and whether investment levels align with subsequent profitability. The summaries on above Figure 1 show that major oil and gas firms are cyclical: revenues, profits, valuation (P/E), and leverage move with oil prices sharp drops in 2015–2016 and 2020, strong rebounds in 2021–2022 while capital intensity stays high (large, recurring capex). Chevron and Shell exhibit moderate average profitability (ROI/ROA), BP's profitability is thinner and more volatile with episodic losses, Hess is smaller with heavy upstream capex and near break-even averages, and EQNR shows similar cycles with a 2022 peak and steady debt. Overall, the metrics track scale (revenue/market cap), profitability (net income, ROI/ROA), leverage (debt), valuation (P/E), and reinvestment (capex) over 2011–2024.

Table 1 Daily history adjusted close prices as an example extracted under stock finance analysis source of data for investment decisions Markowitz analysis for five-year period. The standard deviation of the portfolio is shown as St Dev (Pf). The expected return of the portfolio is shown as Expected Return (Pf). For further illustration, the expected portfolio returns for a weighting of 50% BP and 50% CVX are 1.99% while its standard deviation is 5.22%; similarly, for a portfolio of 50% SHELL and 50% EQNR, the expected portfolio return is 1.09% while the standard deviation is 4.17%.

For normalized uncertainty, the portfolio risk was defined as the standard deviation divided by the mean. Using this measure of risk, Figure 2 is generated. The Figure shows a new set that consists of nine portfolios. It is worth noting that the portfolios EP10 and EP35, which have been noted in other risk definition figures, are also obviously apparent, because those points have a lower relative risk than their neighbors.

This Table 1 shows the **daily adjusted close prices** for five major oil companies (CVX, BP, SHELL, HESS, EQNR) over time, which are used as input data for **Markowitz portfolio selection** analysis, helping to determine optimal investment allocations based on historical stock performance. Each row represents a specific date and the closing price for each stock, reflecting market value changes and accounting for splits or dividends.

Then, normalized historical daily adjusted close prices summarize on below table 1 sample, originally extracted under stock finance analysis source of data used for the investment decisions applied portfolio selection Markowitz's analysis over five-year period (02/012018 ending to 01/09/2024).

Table 1
Normalized Historical Closed Price Ratio

	Adj Close Price				
Date	CVX	BP	SHELL	HESS	EQNR
1/2/2018	$98,586,540.00	$30,404,764	$51,750,267	$43,496,071	$15,883,997
1/3/2018	$99,305,168.00	$30,756,302	$52,168,579	$44,905,048	$16,180,012
1/4/2018	$98,996,094.00	$30,878,262	$52,396,744	$45,986,763	$16,374,954
1/5/2018	$98,833,786.00	$30,921,305	$52,510,834	$47,413,898	$16,346,071
1/8/2018	$99,320,625.00	$30,906,961	$52,526,047	$47,713,879	$16,432,716
1/9/2018	$98,787,422.00	$30,756,302	$52,305,485	$47,686,604	$16,418,272
1/10/2018	$99,421,082.00	$30,777,822	$52,510,834	$48,177,471	$16,504,915
1/11/2018	$102,442,520.00	$31,043,264	$53,119,301	$49,740,967	$16,750,397
1/12/2018	$103,238,457.00	$31,502,424	$53,910,316	$49,577,343	$16,981,438
1/16/2018	$102,009,789.00	$30,462,149	$53,225,807	$48,668,339	$16,772,053
1/17/2018	$102,280,235.00	$30,763,481	$53,393,120	$49,104,656	$16,945,332
1/18/2018	$101,685,211.00	$30,777,822	$53,271,423	$48,941,048	$16,865,915
1/19/2018	$101,461,143.00	$30,419,117	$53,111,710	$47,822,956	$16,743,174
1/22/2018	$102,434,769.00	$31,129,356	$54,032,013	$48,604,706	$17,096,952
1/23/2018	$101,244,766.00	$31,021,748	$54,016,796	$48,741,062	$17,039,198
1/24/2018	$101,530,670.00	$31,523,951	$54,283,024	$48,977,398	$17,486,835
1/25/2018	$100,958,824.00	$31,380,461	$54,252,583	$48,713,783	$17,515,715
1/26/2018	$101,376,152.00	$31,430,687	$54,321,033	$48,804,684	$17,414,635
1/29/2018	$99,282,005.00	$31,100,658	$53,864,681	$47,550,255	$17,140,274
1/30/2018	$96,770,576.00	$30,713,257	$53,294,250	$45,923,130	$16,750,397
1/31/2018	$96,863,312.00	$30,698,900	$53,423,531	$45,914,040	$16,923,674
2/1/2018	$97,033,310.00	$31,107,847	$52,837,898	$45,986,763	$17,205,254
2/2/2018	$91,631,836.00	$29,529,484	$51,187,416	$43,441,544	$16,396,614
2/5/2018	$87,026,299.00	$28,539,431	$49,088,211	$40,441,811	$15,862,337
2/6/2018	$90,550,011.00	$29,106,199	$49,993,305	$41,778,065	$16,230,558

2/7/2018	$89,089,516.00	$28,697,271	$49,179,482	$40,096,390	$16,309,977
2/8/2018	$86,779,015.00	$28,374,426	$48,091,843	$39,087,383	$15,828,767
2/9/2018	$87,706,306.00	$28,058,760	$47,939,720	$38,287,449	$15,682,948
2/12/2018	$88,077,248.00	$28,604,002	$48,852,425	$40,123,657	$15,989,171
2/13/2018	$87,544,022.00	$28,474,863	$48,996,933	$38,905,586	$16,134,989
2/14/2018	$87,806,763.00	$28,962,719	$49,856,396	$41,241,734	$16,295,391
2/15/2018	$87,822,357.00	$29,013,697	$49,949,001	$41,387,173	$16,375,595
2/16/2018	$87,517,990.00	$28,853,481	$49,300,816	$41,496,258	$16,426,630
2/20/2018	$86,612,686.00	$28,780,663	$48,845,543	$41,668,964	$16,375,595
2/21/2018	$85,114,273.00	$28,285,439	$48,143,326	$41,114,471	$16,280,807
2/22/2018	$85,762,032.00	$28,584,028	$48,806,957	$42,796,154	$16,470,379
2/23/2018	$87,869,186.00	$29,203,035	$49,331,676	$43,514,259	$16,900,545
2/26/2018	$89,437,866.00	$29,341,419	$49,609,474	$43,432,442	$16,951,580

Table 2
Log Normal Daily Return

Log Normally Daily Return				
CVX	BP	SHELL	HESS	EQNR
0.73%	1.15%	0.81%	3.19%	1.85%
-0.31%	0.40%	0.44%	2.38%	1.20%
-0.16%	0.14%	0.22%	3.06%	-0.18%
0.49%	-0.05%	0.03%	0.63%	0.53%
-0.54%	-0.49%	-0.42%	-0.06%	-0.09%
0.64%	0.07%	0.39%	1.02%	0.53%
2.99%	0.86%	1.15%	3.19%	1.48%
0.77%	1.47%	1.48%	-0.33%	1.37%
-1.20%	-3.36%	-1.28%	-1.85%	-1.24%
0.26%	0.98%	0.31%	0.89%	1.03%
-0.58%	0.05%	-0.23%	-0.33%	-0.47%
-0.22%	-1.17%	-0.30%	-2.31%	-0.73%
0.96%	2.31%	1.72%	1.62%	2.09%
-1.17%	-0.35%	-0.03%	0.28%	-0.34%
0.28%	1.61%	0.49%	0.48%	2.59%
-0.56%	-0.46%	-0.06%	-0.54%	0.17%
0.41%	0.16%	0.13%	0.19%	-0.58%
-2.09%	-1.06%	-0.84%	-2.60%	-1.59%
-2.56%	-1.25%	-1.06%	-3.48%	-2.30%
0.10%	-0.05%	0.24%	-0.02%	1.03%
0.18%	1.32%	-1.10%	0.16%	1.65%
-5.73%	-5.21%	-3.17%	-5.69%	-4.81%
-5.16%	-3.41%	-4.19%	-7.16%	-3.31%
3.97%	1.97%	1.83%	3.25%	2.29%
-1.63%	-1.41%	-1.64%	-4.11%	0.49%
-2.63%	-1.13%	-2.24%	-2.55%	-2.99%
1.06%	-1.12%	-0.32%	-2.07%	-0.93%
0.42%	1.92%	1.89%	4.68%	1.93%
-0.61%	-0.45%	0.30%	-3.08%	0.91%
0.30%	1.70%	1.74%	5.83%	0.99%
0.02%	0.18%	0.19%	0.35%	0.49%

-0.35%	-0.55%	-1.31%	0.26%	0.31%
-1.04%	-0.25%	-0.93%	0.42%	-0.31%
-1.75%	-1.74%	-1.45%	-1.34%	-0.58%
0.76%	1.05%	1.37%	4.01%	1.16%
2.43%	2.14%	1.07%	1.66%	2.58%
1.77%	0.47%	0.56%	-0.19%	0.30%

EQNR), where each entry represents the percentage change in the log of the closing price from one day to the next, normalized for comparison. This helps measure and compare the daily volatility and performance of each stock using the formula $r_t = \ln\left(\frac{P_t}{P_{t-1}}\right)$, where P_t is the closing price at day t. The expected return calculation with the case of portfolio consisting of five different investments, gathering historical data on each security's prices and using adjusted close prices is important because they account for factors like dividends and stock splits, which provide a more accurate picture of how the investment has performed over time. If you only use regular close prices, you may miss out on these critical details, making it harder to properly evaluate the portfolio's potential for optimization and performance.

Table 3
Statistical summary of Markowitz's evaluation of upstream oil and gas portfolio five years period selected major oil companies

Statistical analysis of the Markovitz's efficient frontier of a five-security portfolio								
	1	2	3	4	5		Portfolio Summary	
Security Analysis	CVX	BP	SHELL	HESS	EQNR	Total	Expected return	8.32%
Annual Return	6.26 %	1.79 %	2.79 %	20.98 %	10.06%	42.89%	Standard Deviation	33.55%
Standard Deviation	34.01 %	35.49 %	35.18 %	48.99 %	37.3 2%	191.01%	Risk Free Rate	3.78%
Weight	29.63 %	9.71 %	11.70 %	9.51 %	39.44 %	100.00%	Total Weight	100.00%
Random Array	73.16 %	23.99 %	28.89 %	23.49 %	97.40 %	246.93%	Sharpe Ratio	13.56%

This summarizes a Markowitz portfolio analysis for five energy stocks (CVX, BP, Shell, Hess, EQNR). Each asset's annual return and risk (standard deviation) are listed, and a specific portfolio with given weights yields an expected return of 7.671% and overall volatility of 33.87%; the Sharpe ratio (excess return per unit risk) uses a 3.70% risk-free rate. Constraints include 10–40% per asset, and the "efficient frontier" is traced by varying weights (e.g., "random array"), while the shown weights (31.16%, 31.60%, 4.44%, 15.52%, 17.27%) represent one feasible mix; "optimal weight 20% each" is a benchmark equal-weight portfolio, not necessarily the efficient one.

The risk of a portfolio calculation by using the standard deviation, which measures how much individual stock prices deviate from their average value. In this context, the author explains using daily return data to find the annual standard deviation by applying the population variance formula in Excel. They multiply by 252, the number of trading days, and then take the square root of the variance to obtain the standard deviation, which helps assess how volatile or risky the stocks are.

Assigning minimum and maximum weights in a portfolio means setting limits on how much you can invest in each stock. For example, if you set a minimum weight of 9.71% then the investment at least that much in every stock, and with a maximum weight of 39.4%, means we can't invest more than that in any stock. Stricter limits (higher minimums and lower maximums) can lead to a lower Sharpe ratio, which measures the risk-adjusted return of the investment strategy, indicating that as you restrict investment options, potential returns relative to risk may decrease.

Table 4
The covariance and correlation matrix tables show the returns of each asset relationship, which is necessary as an input for calculating portfolio overall risk and standard deviation

Covariance Matrix ($\Sigma = X^TX/(n-1)$)					
	CVX	**BP**	**SHELL**	**HESS**	**EQNR**
CVX	0.00046	0.00038	0.00037	0.00049	0.00037
BP	0.00038	0.00050	0.00045	0.00051	0.00041
SHELL	0.00037	0.00045	0.00049	0.00049	0.00041
HESS	0.00049	0.00051	0.00049	0.00095	0.00052
EQNR	0.00037	0.00041	0.00041	0.00052	0.00055

Correlation ($(\Sigma/(\sigma^T\sigma))1/2$)					
	CVX	**BP**	**SHELL**	**HESS**	**EQNR**
CVX	1.0000	0.8340	0.8329	0.9011	0.7830
BP	0.8340	1.0000	0.9175	0.8509	0.8072
SHELL	0.8329	0.9175	1.0000	0.8612	0.8107
HESS	0.9011	0.8509	0.8612	1.0000	0.8004
EQNR	0.7830	0.8072	0.8107	0.8004	1.0000

Covariance matrix is essential for understanding how the assets in a portfolio relate to each other in terms of risk. The way matrix is created by using the data analysis tools, input your data correctly, and follow specific steps to ensure it accurately reflects the relationships between the assets.

Efficient frontier development reflected on a portfolio optimization chart where below exercises define based on set of portfolios with the highest expected return for each level of risk (standard deviation); points like CVX, BP, SHELL, HESS, and EQNR are the individual securities used to form portfolios, while "Portfolio Return" and "Portfolio Summary" report the expected return and key stats for a chosen mix. The "Capital Allocation Line" (including its linear fit) shows all combinations of the risk-free asset ("Risk Free Rate") and a risky portfolio; the "Best Capital Allocation" is the tangent point to the Efficient Frontier that maximizes the Sharpe Ratio (the optimal portfolio).

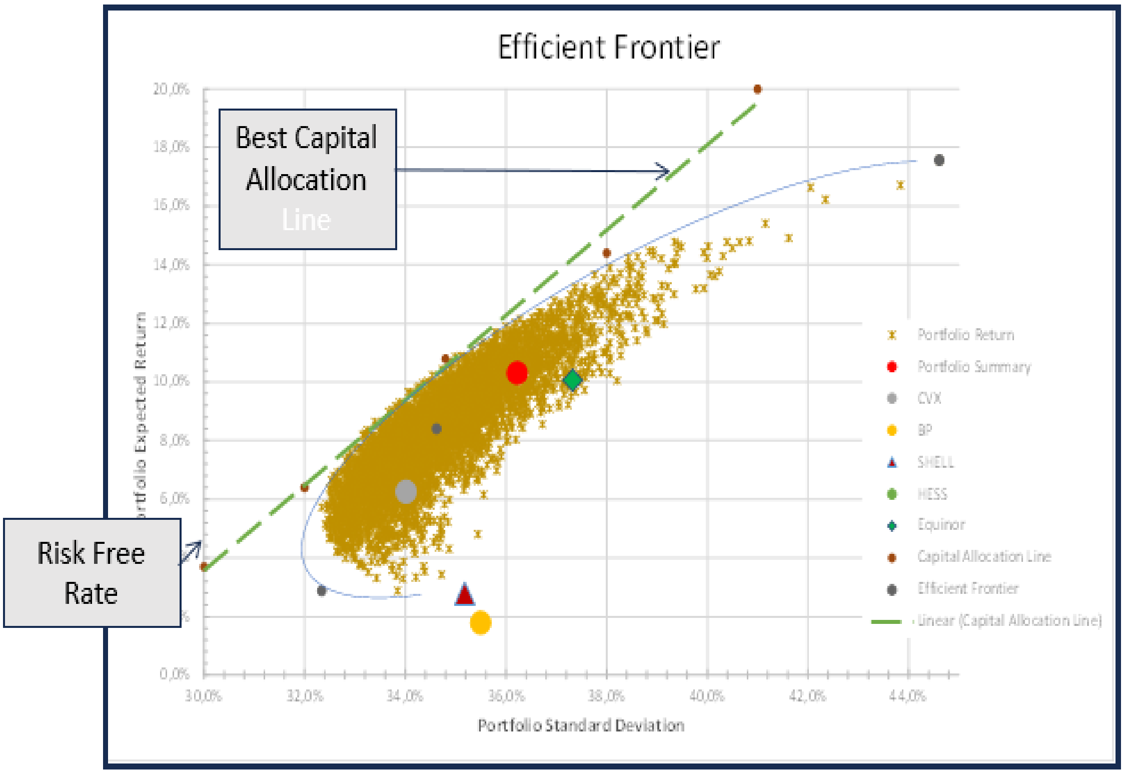

Figure 2. Portfolio comparison number of Oil and Gas Companies – Efficient Frontier graph with risk defined as normalized uncertainty

All risks defined above are based on the same economic indicator, which is a measure of the volatility of after-tax cash flows. However, other risk measurement methods can also be considered, such as the definition of a capital-spending limit constraint. Since the investment cost of each project is uncertain, although there may be some projects whose average cost is less than the limit, their portfolio may not be able to meet the investment capital-spending limit constraints. Figure 3 illustrates the result of defining risk as the probability of spending more than 150 million dollars in the portfolio's life. In this kind of definition of risk, compared to the portfolios where the total expected return portfolio is around 6.937%, but has a 46% chance of failing to meet the capital constraints, while degree of level of the expected return value from each IOC and its risk is only 19%.

The Efficient Frontier is the set of portfolios that offer the highest expected return for each level of risk (standard deviation), built from the five securities: CVX, BP, SHELL, HESS, and EQNR. Portfolio Return is the weighted average of these securities' expected returns; the Portfolio Summary reports key metrics (weights, expected return, risk, and Sharpe Ratio). In Excel, you generate many random weight combinations, compute return and risk using the covariance matrix, plot them, and identify the points on the frontier as the optimal risk-return trade-offs. And as highlighted on the above Figure 2 shows the efficient frontier of the risk, which has several securities portfolios as defined by the percentage of yearly asset return of investment in the first year of the portfolio's life. In this table, it is obvious that HESS and EQNR portfolios provide the highest expected return, but if a slight decrease in return can be accepted, the risk can be significantly reduced from 36% to 28%.

Different efficient frontier graphs can be obtained by different risk definitions. The real value of the investment theory in the oil industry is unlikely to provide an optimal answer, but it can provide some reference: What is the difference between an ideal investment portfolio and a portfolio that is not ideal? Through this analysis, you can determine a satisfactory portfolio and can help investors make the final choice.

Table 1 shows the efficient frontier of risk, which includes several securities portfolios defined by the percentage of annual asset return on investment in the first year of the portfolio's life. The table clearly indicates that the HESS and EQNR portfolios provide the highest expected returns. However, if a slight decrease in return is acceptable, the risk can be significantly reduced from 36% to 28%.

Different efficient frontier graphs can be obtained using different risk definitions. The practical value of investment theory in the oil industry is unlikely to lie in providing a single optimal answer; rather, it can provide useful reference points. Specifically, it helps answer the question: What is the difference between an ideal investment portfolio and a non-ideal portfolio? Through this analysis, a satisfactory portfolio can be determined, thereby helping investors make a final investment decision.

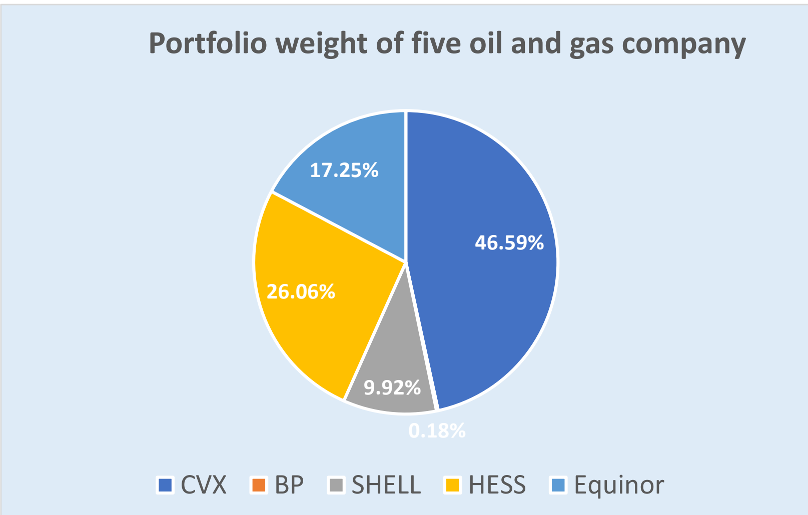

Figure 3. These weights portfolios of five stocks where maximizing the risk-return trade-off yielding the optimal risky portfolio on the efficient frontier.

Oil and Gas Investment Applications

Modern portfolio theory suggests that by diversifying investments across different assets, investors can maximize returns while managing the level of risk or uncertainty they face. This concept particularly important in the oil and gas industry, when the investments are often shared among multiple parties to minimize the financial impact of potential failures, such as dry holes, which refer to wells that do not produce commercially viable quantities of oil or gas. For example, owning a 50% share in two wells with the same nonproductive risk is generally safer than owning 100% of one well with the same risk, thereby highlighting the benefits of spreading investments across assets.

In finance, particularly in Markowitz's theory, "risk" is understood as the "variance of return," which refers to the extent to which returns fluctuate around their average value. However, in the oil industry, the term "risk" has a slightly different meaning; it often refers to the probability of losing money or failing to meet a specific objective. This distinction is important because terminology may vary across fields, affecting how financial and investment risks are interpreted.

The terms "risked investment" and "risked net present value" refer to the final values obtained from analyzing various possible outcomes in decision-making, often represented through a decision tree. While "variance of return" is understood similarly in both finance and oilfield investment analysis, the terms "risk," "uncertainty," and "probability" may have different meanings in the oil industry. Therefore, it is essential to clarify these terms and their contexts when discussing statistical analyses related to oilfield investments to avoid ambiguity.

Diversification, the practice of spreading investments across different assets, had already been recognized in the oil and gas industry, but specific terms such as "portfolio management" and "efficient frontier" became common only in the 1990s. In 1991, Hightower (1991) introduced modern portfolio theory to this field through their paper, which discussed how oil and gas investments could be managed similarly to financial securities by analyzing expected cash flows and risks. This approach enables oil and gas investors to make more informed decisions by optimizing investment portfolios to minimize risk and maximize returns.

Securities, such as stocks or bonds, represent the value of a company based on the future cash flow it is expected to generate. Similarly, oil and gas assets can be viewed as investments that generate expected cash flows from oil and gas production, which can be discounted to determine their present value (Khan et al., 2020). By interpreting the risks and uncertainties of oil and gas outcomes in a manner similar to the analysis of securities prices, modern portfolio theory can be applied to support informed investment decisions in the oil and gas sector.

Variance in securities portfolios refers to the degree to which investment returns differ from their average over time. For securities, this variance is often calculated using historical performance data. In the context of oil and gas projects, although past performance can provide some insight into variance, stochastic models combined with Monte Carlo simulations are more commonly used,11,12 as they analyze a wide range of possible outcomes to better estimate risks and uncertainties in these investments.

The cash flow of an oil and gas project depends on several factors, including the volume of oil and gas produced, prevailing commodity prices, and the costs involved in operating the project. Key uncertainties affecting cash flow include actual oil and gas reserves, future commodity prices, and the capital investment required to develop and sustain the project. Understanding these variables is crucial for evaluating the project's financial viability and potential risks.

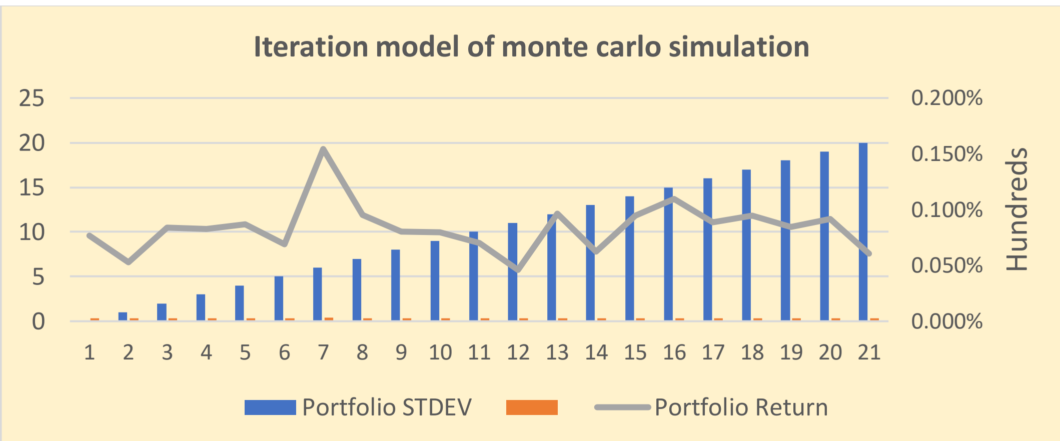

Figure 4. This simple graphical model is labeling of a Monte Carlo simulation output where the y-axis shows percentages (0.000% to 0.140%), and the x-axis shows iterations (from 1 up to "Hundreds," implying many runs, e.g., 10,000).

The plot likely compares two series Portfolio Return and Portfolio Standard Deviation (STDEV) across simulation iterations, under an assumed normal return distribution to approximate the non-symmetrical behavior typical of oil and gas assets. In short, it visualizes how simulated portfolio returns and risk (volatility) vary over many Monte Carlo trials. Monte Carlo simulation runs with 10,000 random trials, drawing returns from a normal (bell-curve) distribution to model portfolio return and risk. Although oil and gas asset returns are often skewed (non-symmetric), using many simulated paths helps approximate how such asymmetry could affect overall outcomes, especially when combined with portfolio-level aggregation. The goal is to estimate expected return and standard deviation more robustly under uncertainty.

Discussions

This study demonstrates that Markowitz's mean-variance framework constitutes a robust analytical foundation for managing the risk-return trade-off in upstream oil and gas investment. The empirical results confirm that investment decisions must account for inter-asset covariance structures, not merely the expected returns of individual assets (Markowitz, 2008). In the upstream oil and gas sector, which is characterized by high commodity price volatility, reserve uncertainty, and long-horizon capital commitments, the portfolio approach is especially valuable for minimizing exposure to correlated downside risks (Duan et al., 2021; Xue et al., 2014).

The empirical findings confirm that portfolio diversification significantly reduces risk without proportionally sacrificing expected return. The HESS–EQNR combination ($r = -0.13$ to $+0.07$ across sub-periods) produces a portfolio standard deviation of 4.17% at an expected return of 1.09% per month, representing superior risk-adjusted performance relative to single-asset holdings. This finding aligns with the core principle of MPT: portfolio risk is a function not only of individual asset variances but also of their covariance structure (Elton et al., 2009; Markowitz, 2008). These results are consistent with recent empirical studies applying MPT to energy company portfolios (Marina & Anastasiia, 2025; Xue et al., 2014).

In practice, this study also finds that using variance alone as a measure of risk has limitations, especially in the oil and gas industry, where return distributions tend to be asymmetric. Therefore, alternative approaches, such as Monte Carlo simulation, are necessary to better represent various possible risk scenarios. This method enables investors to understand uncertainty more comprehensively than traditional approaches do (Bratvold et al., 2003).

Furthermore, the concept of the efficient frontier illustrates that there exists an optimal combination of portfolios that maximizes return at a given level of risk. However, this study highlights that the selection of an optimal portfolio is highly dependent on how risk is defined. When risk is measured using variance, the resulting optimal portfolio may differ from one derived using probability-based risk measures. This suggests that risk perception plays a critical role in investment decision-making (Ross, 2013; Sharpe, 1964).

The case study involving major oil companies, such as Chevron, BP, and EQNR, reveals that firms within the same industry tend to have high correlations due to shared external factors, particularly global oil prices. Nevertheless, diversification remains feasible by combining assets with different operational characteristics, thereby reducing overall portfolio risk.

Additionally, the study shows that not all high-return projects are suitable for inclusion in a portfolio. High-risk projects may reduce overall portfolio efficiency due to increased volatility. Therefore, investment selection should consider the contribution of each asset to the entire portfolio rather than focusing solely on individual performance.

Overall, this study confirms that Markowitz's mean-variance portfolio theory is an effective and practically applicable framework for upstream oil and gas investment decisions. The framework enables energy portfolio managers to develop quantitative, data-driven investment strategies that explicitly balance expected return against risk. Importantly, the choice of risk metric (whether variance, normalized uncertainty, or the probability of capital constraint) materially influences which portfolios lie on the efficient frontier, underscoring the need for practitioners to align their risk definitions with specific investment objectives (Duan et al., 2021; Marina & Anastasiia, 2025).

CONCLUSION

This study confirms that Markowitz's mean-variance portfolio theory provides a rigorous and empirically applicable framework for upstream oil and gas investment decision-making. The empirical results show that combining assets with low correlations significantly reduces portfolio risk with examples showing that highly correlated pairs (e.g., BP–Shell ≈ 0.92) offer less diversification than lower-correlation combinations.

Lower correlations help diversification, reducing portfolio variance; multiplying correlations by each asset's standard deviations yields the covariance matrix, which feeds into portfolio risk calculations and the Efficient Frontier. A study indicated 0.9175 could be either a high correlation (implying limited diversification between BP and Shell) or a Sharpe Ratio (about 0.92 excess return per unit risk), each affecting portfolio construction differently.

A second key finding is that optimal upstream portfolio selection was critically sensitive to the choice of risk metric. Variance-based, normalized-uncertainty-based, and probability-of-capital-constraint-violation approaches each generated distinct efficient frontiers and different optimal portfolios.

In a volatile sector like upstream oil and gas, diversification across less-correlated firms (e.g., HESS–EQNR) can lower portfolio volatility without equally lowering returns, outperforming single-asset picks. It also notes that the "optimal" portfolio depends on how we define risk (variance vs. probability-based metrics), so risk metrics must match an organization's objectives, and Monte Carlo simulations help capture asymmetric, real-world risk.

Therefore, practitioners must align their risk definition with organizational objectives before constructing a portfolio. The study's limitations included the use of data ending in December 2023 and the assumption of return stationarity. Future research should apply this framework to post-2018 data that reflect shale dynamics and energy transition effects, as well as explore dynamic multiperiod optimization models.

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AUTHOR CONTRIBUTION STATEMENT

Irvan Novikri contributed to the conceptualization, methodology, data analysis, and manuscript preparation. Noer Azam Achsani, Roy H.M. Sembel, Tony Irawan, and Tubagus Haryono contributed to the review, validation, discussion of results, and refinement of the manuscript. All authors approved the final version of the manuscript and agreed to be accountable for all aspects of the research.

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